CS420 Final Preparation

Department of Computer Science

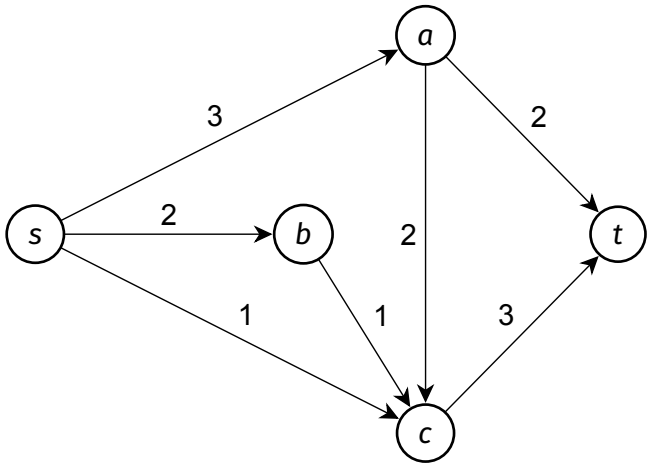
Spring 2026

Instructions You may use these questions as a study plan for the final.

You can prepare a cheat sheet (max 1 side of US letter paper) to help you study and for reference during the final exam.

The final may have some questions very similar to these ones and some completely different.

1. Consider the flow network pictured below.



   (a) Use something like the Ford–Fulkerson algorithm to find a maximum flow in G.

   (b) Find a cut with cost equal to the maximum flow.

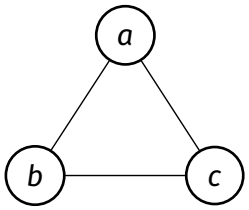
   (c) Learn the statement and proof of the max-flow min-cut theorem.

2. (a) Recall an algorithm that finds the maximum matching in a bipartite graph.

   (b) Use it to find the maximum matching in a small example graph on, say, 7 vertices.

   (c) Why does the algorithm work? Can we **prove** that if the matching is not maximum then there must be an augmenting path?

3. Consider the triangle graph $G = (V, E)$ with $V = \{a, b, c\}$ and $E = \{ab, bc, ac\}$ pictured below.



   (a) What does the integer program

$$\max \sum_{v \in V} x_v \text{ such that}$$
$$x \in \{0, 1\}^V,$$
$$\forall uv \in E, x_u + x_v \leq 1$$

   compute in G?

   (b) Consider the fractional relaxation of the program, allowing $x \in [0, 1]^V$ but keeping the same objective and constraints. Show that there is a feasible point with value strictly greater than 1.

4. Write down an integer program that computes the max cut of a graph.

5. Familiarize yourself with the proof of the Access Lemma of splay trees and answer the following questions.

   (a) For each of the three splay steps, why is it enough to prove that the amortized cost of the step is
   - $\leq 3(r'(x) - r(x)) + 1$ for the zig
   - $\leq 3(r'(x) - r(x))$ for the zigzig
   - $\leq 3(r'(x) - r(x))$ for the zigzag

   (b) Make sure you know where the facts such as $r'(x) = r(z)$ come from.

   (c) Practice the algebra which lets you see how to apply the concavity of the logarithm.

[END]